

HOW TO FIND THE QCD CRITICAL POINT^a

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March 30, 1999. MIT-CTP-2848

The event-by-event fluctuations in heavy ion collisions carry information about the thermodynamic properties of the hadronic system at the time of freeze-out. By studying these fluctuations as a function of varying control parameters, such as the collision energy, it is possible to learn much about the phase diagram of QCD. As a timely example, we stress the methods by which present experiments at the CERN SPS can locate the second order critical point at which a line of first order phase transitions ends. Those event-by-event signatures which are characteristic of freeze-out in the vicinity of the critical point will exhibit nonmonotonic dependence on control parameters. We focus on observables constructed from the multiplicity and transverse momenta of charged pions. We find good agreement between NA49 data and thermodynamic predictions for the noncritical fluctuations of such observables. We then analyze the effects due to the critical fluctuations of the sigma field. We estimate the size of these nonmonotonic effects which appear near the critical point, including restrictions imposed by finite size and finite time.

1 Introduction

In my talk at Strong and Electroweak Matter '98, I presented recent work on the physics which arises in two different areas of the QCD phase diagram. Cold dense quark matter forms a color superconductor, and I compared the superconducting phase expected in QCD with two massless quarks with that expected for three massless quarks in which chiral symmetry is broken by color-flavor locking. Alford, Berges and I have recently completed an analysis of the phase diagram of zero temperature QCD as a function of density and strange quark mass, in Ref. ². This brings the ideas I presented in Copenhagen together into a consistent picture, and I refer you to that paper for an up-to-date treatment of the subject, and references to the literature.

The other half of my talk in Copenhagen was a sketch of methods by which present heavy ion experiments can find the critical point in the QCD phase diagram at nonzero temperature T and baryon chemical potential μ . Like the end point in the electroweak phase diagram, discussed by others at this meeting, this critical point is a second order transition in the Ising universality class which occurs at the end of a line of first order phase transitions. Stephanov,

^aWork done in collaboration with Misha Stephanov and Edward Shuryak.¹

Shuryak and I have recently completed a detailed analysis of the signatures of the physics characteristic of the vicinity of this point,¹ begun in Ref. ³. I described this work in a preliminary fashion in Copenhagen; in these proceedings, I summarize the results and implications of Ref. ¹. Those interested in the derivation of these results should see Ref. ¹.

Large acceptance detectors, such as NA49 and WA98 at CERN, have made it possible to measure important average quantities in single heavy ion collision events. For example, instead of analyzing the distribution of charged particle transverse momenta obtained by averaging over particles from many events, we can now study the event-by-event variation of the mean transverse momentum of the charged pions in a single event, p_T .^b Although much of this data still has preliminary status, with more statistics and more detailed analysis yet to come, some general features have already been demonstrated. In particular, the event-by-event distributions of these observables are as perfect Gaussians as the data statistics allow, and the fluctuations — the width of the Gaussians — are small.⁴

This is very different from what one observes in pp collisions, in which fluctuations are large. These large non-Gaussian fluctuations clearly reflect non-trivial quantum fluctuations, all the way from the nucleon wave function to that of the secondary hadrons, and are not yet sufficiently well understood. As discussed in Refs. ^{5,6}, thermal equilibration in AA collisions drives the variance of the event-by-event fluctuations down, close to the value determined by the variance of the inclusive one-particle distribution divided by the square root of the multiplicity.

Can we learn something from the magnitude of these small fluctuations and their dependence on the parameters of the collision? What do the widths of the Gaussians tell us about the thermodynamics of QCD? Some of these questions have been addressed in Refs. ^{7,8} where it was pointed out that, for example, temperature fluctuations are related to heat capacity via

$$\frac{\langle(\Delta T)^2\rangle}{T^2} = \frac{1}{C_V(T)}, \quad (1)$$

and so can tell us about thermodynamic properties of the matter at freeze-out. Furthermore, Mrówczyński has discussed the study of the compressibility of hadronic matter at freeze-out via the event-by-event fluctuations of the particle number⁹ and Gaździcki¹⁰ and Mrówczyński¹¹ have considered event-by-event fluctuations of the kaon to pion ratio as measured by NA49⁴. In pp physics one can hope to extract quantum mechanical information about the initial

^bWe denote the mean transverse momentum of all the pions in a single event by p_T rather than $\langle p_T \rangle$ because we choose to reserve $\langle \dots \rangle$ for averaging over an ensemble of events.

state from event-by-event fluctuations of the final state; in heavy ion collisions equilibration renders this an impossible goal. In AA collisions, then, the new goal is to use the much smaller, Gaussian event-by-event fluctuations of the final state to learn about thermodynamic properties at freeze-out.

It is worth noting that once a large acceptance detector has presented convincing evidence that the event-by-event distribution of, for example, p_T is Gaussian, then the measurement of the width of such a distribution can be accomplished by “event-by-event” measurements in which only two pions per event are observed. This has recently been emphasized by Białas and Koch.¹² Of course, this approach measures the width of the event-by-event distribution whether or not it is Gaussian; it is only the results of a large acceptance experiment like NA49 which motivate a thermodynamic analysis of the event-by-event fluctuations.

Stephanov, Shuryak and I focus on observables constructed from the multiplicity and the momenta of the charged particles in the final state, as measured by NA49. We leave the extension of the methods of this paper to the study of thermodynamic implications of the NA49 Gaussian distribution of event-by-event K/π ratios⁴ and of the WA98 Gaussian distribution of event-by-event π^0/π^\pm ratios¹³ for future work.

One of the lessons of our paper is that it is difficult to apply thermodynamic relations like (1) directly. To see a sign of this, note that the event-by-event fluctuations of the energy E of a part of a finite system in thermal equilibrium are given by $\langle(\Delta E)^2\rangle = T^2 C_V(T)$. For a system in equilibrium, the mean values of T and E are directly related by an equation of state $E(T)$; their fluctuations, however, have quite different behavior as a function of C_V , and therefore behave differently when C_V diverges at a critical point. The fluctuations of “mechanical” observables increase at the critical point. Because $T(E)$ is singular at the critical point, the fluctuations of T decrease there. It is a fact that what we measure are the mechanical observables, and since we in general only know $T(E)$ for simple systems we call thermometers, we cannot apply (1) to the complicated system of interest. It is not in fact necessary to translate the observed “mechanical” variable (the mean transverse momentum p_T for example) into a temperature in order to detect the critical point. It is easier to look directly at the fluctuations of observable quantities. We demonstrate that the fluctuations of p_T grow at the critical point.

Although our methods are general, we focus in Ref. ¹ on how to use them to find and study the critical end-point E on the phase diagram of QCD in the $T\mu$ plane. The possible existence of such a point, as an endpoint of the first order transition separating quark-gluon plasma from hadron matter, and its universal critical properties have been pointed out recently in Refs. ^{14,15}.

The point E can be thought of as a descendant of a tricritical point in the phase diagram for 2-flavor QCD with *massless* quarks. In a previous letter, we have laid out the basic ideas for observing the critical endpoint³. The signatures proposed in Ref.³ are based on the fact that such a point is a genuine thermodynamic singularity at which susceptibilities diverge and the order parameter fluctuates on long wavelengths. The resulting signatures all share one common property: they are *nonmonotonic* as a function of an experimentally varied parameter such as the collision energy, centrality, rapidity or ion size. Once experimentalists vary a control parameter which causes the freeze-out point in the (T, μ) plane to move toward, through, and then past the vicinity of the endpoint E, they should see all the signatures we describe first strengthen, reach a maximum, and then decrease, as a nonmonotonic function of the control parameter. It is important to have a control parameter whose variation changes the μ at which the system crosses the transition region and freezes out. The collision energy is an obvious choice, since it is known experimentally that varying the collision energy has a large effect on μ at freeze-out. Other possibilities should also be explored.^c

We assume throughout that freeze-out occurs from an equilibrated hadronic system. If freeze-out occurs “to the left” (lower μ ; higher collision energy) of the critical end point E, it occurs after the matter has traversed the crossover region in the phase diagram. If it occurs “to the right” of E, it occurs after the matter has traversed the first order phase transition. This is the situation in which our assumption of freeze-out from an equilibrated system is most open to question. First, one may imagine hadronization directly from the mixed phase, without time for the hadrons to rescatter. Hadronic elastic scattering cross-sections are large enough that this is unlikely. Second, one may worry that the matter is inhomogeneous after the first order transition, and has not had time to re-equilibrate. Fortunately, our assumption is testable. If the matter were inhomogeneous at freeze-out, one can expect non-Gaussian fluctuations in various observables¹⁶ which would be seen in the same experiments that seek the signatures we describe. We focus on the Gaussian thermal fluctuations of an equilibrated system, and study the nonmonotonic changes in these fluctuations associated with moving the freeze-out point toward and then past the critical point, for example from left to right as the collision energy is reduced.

Ref.¹ is devoted to a detailed analysis of the physics behind event-by-event fluctuations in relativistic heavy ion collisions and the resulting effects unique to the vicinity of the critical point in the phase diagram of QCD. Most of

^cIf the system crosses the transition region near E, but only freezes out at a much lower temperature, the event-by-event fluctuations will not reflect the thermodynamics near E. In this case, one can push freeze-out to earlier times and thus closer to E by using smaller ions.³

our analysis is applied to the fluctuations of the observables characterizing the multiplicity and momenta of the charged pions in the final state of a heavy ion collision. There are several reasons why the pion observables are most sensitive to the critical fluctuations. First, the pions are the most numerous hadrons produced and observed in relativistic heavy ion collisions. A second, very important reason, is that pions couple strongly to the fluctuations of the sigma field (the magnitude of the chiral condensate) which is the order parameter of the phase transition. Indeed, the pions are the quantized oscillations of the phase of the chiral condensate and so it is not surprising that at the critical end point, where the magnitude of the condensate is fluctuating wildly, signatures are imprinted on the pions.

2 Noncritical Thermal Fluctuations in Heavy Ion Collisions

Before we discuss the effects of the critical fluctuations, we must analyze the thermal fluctuations which are present if freeze-out does *not* occur in the vicinity of the critical point. In this section, but not throughout this paper, we assume that the system freezes out far from the critical point in the phase diagram, and can be approximated as an ideal resonance gas when it freezes out. We compare some of our results to preliminary data from the NA49 experiment on PbPb collisions at 160 AGeV, and find broad agreement. The results obtained seem to support the hypotheses that most of the fluctuation observed in the data is indeed thermodynamic in origin, and that this system is not freezing out near the critical point.

As a first test of our resonance gas model, we analyze the fluctuations in an ideal Bose gas of pions, and then add as many of the effects which this simple treatment neglects except that we assume that no effects due to critical fluctuations are significant. We model the matter in a relativistic heavy ion collision at freeze-out as a resonance gas in thermal equilibrium, and begin by calculating the variance of the event-by-event fluctuations of total multiplicity N . The fluctuations in N are not affected by the boost which the pion momenta receive from the collective flow, but they are contaminated experimentally by fluctuations in the impact parameter. This experimental contamination can be reduced by making a tight enough centrality cut using a zero degree calorimeter.

We find $\langle(\Delta N)^2\rangle/\langle N\rangle \approx 1.5$, which we compare with NA49 results from central Pb-Pb collisions at 160 AGeV. It is clear that with no cut on centrality, one would see a very wide non-Gaussian distribution of multiplicity determined by the geometric probability of different impact parameters b . Gaussian thermodynamic fluctuations can only be seen if a tight enough cut in centrality

is applied. The event-by-event N -distribution found by NA49 when they use only the 5% most central of all events, with centrality measured using a zero degree calorimeter, is Gaussian to within about 5%. This cut corresponds to keeping collisions with impact parameters $b < 3.5$ fm.⁴ The non-Gaussianity could be further reduced by tightening the centrality cut further. From the data, we have $\langle(\Delta N)^2\rangle/\langle N\rangle = 2.008 \pm 0.009$, which suggests that about 75% of the observed fluctuation is thermodynamic in origin. The contamination introduced into the data by fluctuations in centrality could be reduced by analyzing data samples with more or less restrictive cuts but the same $\langle N\rangle$, and extrapolating to a limit in which the cut is extremely restrictive. This could be done using cuts centered at any centrality. Our resonance gas model predicts that as the centrality cut is tightened, the ratio $v_{\text{ebe}}^2(N)/\langle N\rangle$ should decrease toward a limit near 1.5.

Although further work is certainly required, it is already apparent that the bulk of the multiplicity fluctuations observed in the data are thermodynamic in origin. impact parameter. Note that our prediction is strongly dependent on the presence of the resonances; had we not included them, our prediction would have been significantly lower, farther below the data. Because the multiplicity fluctuations are sensitive to impact parameter fluctuations, it may prove difficult to explain their magnitude with greater precision even in future. However, the fact that they are largely thermodynamic in origin suggests that the effects present near the critical point, which we describe below, could result in a significant nonmonotonic enhancement of the multiplicity fluctuations. This would be of interest whether or not the noncritical fluctuations on top of which the nonmonotonic variation occurs are understood with precision.

We then turn to a calculation of the variance of the event-by-event fluctuations of the mean transverse momentum, p_T . We first calculate the width of the *inclusive* p_T -distribution, $v_{\text{inc}}(p_T)$. In the absence of any correlations, the event-by-event fluctuations of the mean transverse momentum of the charged pions in an event, $v_{\text{ebe}}(p_T) \equiv \langle(\Delta p_T)^2\rangle^{1/2}$ would be given by $v_{\text{inc}}(p_T)/\langle N\rangle^{1/2}$, and this turns out to be a very good approximation in the present data as we discuss below. We calculate numerically the contribution to $v_{\text{inc}}(p_T)$ from “direct pions”, already present at freeze-out, and from the pions generated later by resonance decay. We have simulated a gas of pions, nucleons and resonances in thermal equilibrium at freeze-out, including the π , K , η , ρ , ω , η' , N , Δ , Λ , Σ and Ξ , and then simulated the subsequent decay of the resonances. That is, we have generated an ensemble of pions in three steps: (i) Thermal ratios of hadron multiplicities were calculated assuming equilibrium ratios at chemical freeze-out. Following¹⁷, the values $T_{\text{ch}} = 170$ MeV and $\mu_{\text{baryon}} = 200$ MeV were used. (ii) Then, a program generates hadrons with multiplicities

determined at chemical freezeout, but with thermal momenta as appropriate at the thermal freeze-out temperature, which we take to be $T_f = 120$ MeV, with $\mu_\pi = 60$ MeV. The last step (iii) is to decay all the resonances. From the resulting ensemble of pions (the sum of the direct pions and those from the resonances) we obtain $v_{\text{inc}}(p_T)/\langle p_T \rangle = 0.66$. The resonances turn out to be less important here than in the calculation of the multiplicity fluctuations, in that the resonance gas prediction for $v_{\text{inc}}(p_T)/\langle p_T \rangle$ is almost indistinguishable from that of an ideal Bose gas of pions.

To this point, we have calculated the fluctuations in p_T as if the matter in a heavy ion collision were at rest at freeze-out. This is not the case: by that stage the hadronic matter is undergoing a collective hydrodynamic expansion in the transverse direction, and this must be taken into account in order to compare our results with the data. A very important point here is that the fluctuations in pion multiplicity are not affected by flow, and our prediction for them is therefore unmodified. However the event-by-event fluctuations of mean p_T are certainly affected by flow. The fluctuations we have calculated pertain to the rest frame of the matter at freeze-out, and we must now boost them. A detailed account of the resulting effects would require a complicated analysis. We use the simple approximation¹⁹ that the effects of flow on the pion momenta can be treated as a Doppler blue shift of the spectrum: $n(p_T) \rightarrow n(p_T \sqrt{1-\beta}/\sqrt{1+\beta})$. This blue shift increases $\langle p_T \rangle$, and increases $v_{\text{inc}}(p_T)$, but leaves the ratio $v_{\text{inc}}(p_T)/\langle p_T \rangle$ (and therefore the ratio $v_{\text{ebe}}(p_T)/\langle p_T \rangle$) unaffected. However, event-by-event fluctuations in the flow velocity β must still be taken into account. This issue was discussed qualitatively already in⁸, where it was argued that this effect must be relatively weak. In Ref. ¹ we provide the first rough estimate of its magnitude. We estimate that fluctuations in the flow velocity increase $v_{\text{inc}}(p_T)/\langle p_T \rangle$ from 0.66 to 0.67. The largest uncertainty in our estimate for $v_{\text{inc}}(p_T)/\langle p_T \rangle$ is not due to the fluctuations in the flow velocity, which can clearly be neglected, but is due to the velocity itself. The blue shift approximation which we have used applies quantitatively only to pions with momenta greater than their mass¹⁹. Because of the nonzero pion mass, boosting the pions does not actually scale the momentum spectrum by a momentum independent factor. Furthermore, in a real heavy ion collision there will be a position dependent profile of velocities, rather than a single velocity β . A more complete calculation of $v_{\text{inc}}(p_T)/\langle p_T \rangle$ would require a better treatment of these effects in a hydrodynamic model; we leave this for the future.

We compare our results to the NA49 data, in which $v_{\text{inc}}(p_T)/\langle p_T \rangle = 0.749 \pm 0.001$. We see that the major part of the observed fluctuation in p_T is accounted for by the thermodynamic fluctuations we have considered. Our

prediction is about 10% lower than that in the data. First, this suggests that there may be a small nonthermodynamic contribution to the p_T -fluctuations, for example from fluctuations in the impact parameter. (However, we expect that the fluctuations of an intensive quantity like p_T are less sensitive to impact parameter fluctuations than are those of the multiplicity, and this seems to be borne out by the data.) The other source of the discrepancy is the blue shift approximation. We leave a more sophisticated treatment of the effects of flow on the spectrum to future work. Such a treatment is necessary before we can estimate how much of the 10% discrepancy is introduced by the blue shift approximation. Future work on the experimental side (varying the centrality cut) could lead to an estimate of how much of the discrepancy is due to impact parameter fluctuations.

We have gone as far as we will go in this paper in our quest to understand the thermodynamic origins of the width of the inclusive single particle distribution. We now turn to the ratio of the scaled event-by-event variation to the variance of the inclusive distribution:

$$\sqrt{F} \equiv \frac{\langle N \rangle^{1/2} v_{\text{ebe}}(p_T)}{v_{\text{inc}}(p_T)} = 1.002 \pm 0.002. \quad (2)$$

The difference between the scaled event-by-event variance and the variance of the inclusive distribution is less than a percent in the NA49 data.^d

We analyze a number of noncritical contributions to the ratio \sqrt{F} , which we write

$$\sqrt{F} = \sqrt{F_B F_{\text{res}} F_{\text{EC}}} . \quad (3)$$

F_B is the contribution of the Bose enhancement of the fluctuations of identical pions. We calculate this effect and find $\sqrt{F_B} \approx 1.02$. F_{res} describes the effect of the correlations induced by the fact that pions produced by the decay of a resonance after freeze-out do not have a chance to rescatter. We estimate it by dividing the pions from our resonance gas simulation into “events” of varying sizes, and evaluating F . Since Bose enhancement is not included in the

^dWe explain in an Appendix in Ref. ¹ that in order to be sure that $F = 1$ when there are no correlations between pions, care must be taken in constructing an estimator for $v_{\text{ebe}}(p_T)$ using a finite sample of events, each of which has finite multiplicity. The appropriate prescription is to weight events in the event-by-event average by their multiplicity, and we have made the appropriate correction in writing (2). Other authors ⁵ have introduced the correlation measure $\Phi_{p_T} = \langle N \rangle^{1/2} v_{\text{ebe}}(p_T) - v_{\text{inc}}(p_T)$. Because $v_{\text{inc}}(p_T)$ is scaled by the blue shift introduced by the expansion velocity, so is Φ_{p_T} . This makes Φ_{p_T} harder to predict than F . However, for convenience, we note that if one uses the experimental value of $v_{\text{inc}}(p_T)$, a value $\sqrt{F} = 1.01$ corresponds to $\Phi_{p_T} = 2.82$ MeV, and the \sqrt{F} in the data (2) corresponds to $\Phi_{p_T} = 0.6 \pm 0.6$ MeV.

simulation, the F so obtained is just F_{res} . We find no statistically significant contribution, and conclude that $|F_{\text{res}} - 1| < 0.01$.

The third contribution, F_{EC} , is due to energy conservation in a finite system. This is most easily described by considering the event-by-event fluctuations Δn_p in the number of pions in a bin in momentum space centered at momentum p . Consider the correlator $\langle \Delta n_p \Delta n_k \rangle$. When one n_p fluctuates up, others must fluctuate down, and it is therefore more likely that n_k fluctuates downward. Energy conservation in a finite system therefore leads to an anti-correlation which is off-diagonal in pk space. $v_{\text{ebe}}(p_T)$ is determined by $\langle \Delta n_p \Delta n_k \rangle$, and the result of this anti-correlation is a reduction:

$$\sqrt{F_{\text{EC}}} \approx 0.99. \quad (4)$$

If the observed charged pions are in thermal contact with an unobserved heat bath, the anti-correlation introduced by energy conservation decreases as the heat capacity of the heat bath increases. The estimate (4) assumes that the heat capacity of the direct charged pions is about 1/4 of the total heat capacity of the hadronic system at freezeout. In addition to the contributions we calculate, \sqrt{F} is affected by the finite two-track resolution in the detector, and by final state Coulomb interactions between charged pions. NA49 estimates that these contributions reduce F by about the same amount that Bose enhancement increases it.

We conclude that the ratio \sqrt{F} measured by NA49 is broadly consistent with thermodynamic expectations. It receives a positive contribution from Bose enhancement, negative contributions from energy conservation and two-track resolution, and a positive contribution from the effect of resonance decays. These contributions to \sqrt{F} are all roughly at the 1% level (or smaller in the case of that from resonance decays) and it seems that they cancel in the data (2). Our results support the general idea that the small fluctuations observed in AA collisions, relative to those in pp , are consistent with the hypothesis that the matter in the AA collisions achieves approximate local thermal equilibrium in the form of a resonance gas.

With more detailed experimental study, either now at the SPS, or soon at RHIC (STAR will study event-by-event fluctuations in p_T , N , particle ratios, etc; PHENIX and PHOBOS in N only) it should be possible to disentangle the different effects we describe. Making a cut to look at only low p_T pions should increase the effects of Bose enhancement. The anti-correlation introduced by energy conservation is due to terms in $\langle \Delta n_p \Delta n_k \rangle$ which are off-diagonal in pk . Thus, a direct measurement of $\langle \Delta n_p \Delta n_k \rangle$ would make it easy to separate this anti-correlation from other effects. The cross correlation $\langle \Delta N \Delta p_T \rangle$ is also a very interesting observable to study. It vanishes for a classical ideal

gas. This means that whereas $v_{\text{ebe}}(p_T)$ receives a dominant contribution from the width of the inclusive single particle distribution, this effect cancels in $\langle \Delta N \Delta p_T \rangle$ and the remaining effects due to Bose enhancement and energy conservation dominate. Although this cross-correlation is small, it is worth measuring because it only receives contributions from interesting effects.

We hope that the combination of the theoretical tools we have provided and the present NA49 data provide a solid foundation for the future study of the thermodynamics of the hadronic matter present at freeze-out in heavy ion collisions. Once data is available for other collision energies, centralities or ion sizes, the present NA49 data and the calculations of this section will provide an experimental and a theoretical baseline for the study of variation as a function of control parameters.

Our analysis demonstrates that the observed fluctuations are broadly consistent with thermodynamic expectations, and therefore raises the possibility of large effects when control parameters are changed in such a way that thermodynamic properties are changed significantly, as at a critical point. The smallness of the statistical errors in the data also highlights the possibility that many of the interesting systematic effects we analyze in this paper will be accessible to detailed study as control parameters are varied.

3 Pions Near the Critical Point: Interaction with the Sigma Field

With the foundations established, we now describe how the fluctuations we analyze will change if control parameters are varied in such a way that the baryon chemical potential at freeze-out, μ_f , moves toward and then past the critical point in the QCD phase diagram at which a line of first order transitions ends at a second order endpoint. The good agreement between the noncritical thermodynamic fluctuations we analyze in Section 2 and NA49 data make it unlikely that central PbPb collisions at 160 AGeV freeze out near the critical point. Estimates we have made in Ref. ³ suggest that the critical point is located at a baryon chemical potential μ such that it will be found at an energy between 160 AGeV and AGS energies. This makes it a prime target for detailed study at the CERN SPS by comparing data taken at 40 AGeV, 160 AGeV, and in between. If the critical point is located at such a low μ that the maximum SPS energy is insufficient to reach it, it would then be in a regime accessible to study by the RHIC experiments. We want to stress that we are more confident in our ability to describe the properties of the critical point and thus to predict *how* to find it than we are in our ability to predict where it is.

We now describe how the fluctuations of the pions will be affected if the

system freezes out near the critical endpoint. First, because the pions at freeze-out are now in contact with a heat bath whose heat capacity diverges at the critical point, the effects of energy conservation parametrized by $F_{\text{EC}} - 1$ are greatly reduced. However, since F_{EC} is close to one even away from the critical point, this is a small effect.

The dominant effects of the critical fluctuations on the pions are the direct effects occurring via the $\sigma\pi\pi$ coupling. In the previous section, we made the assumption that the “direct pions” at freeze-out could be described as an ideal Bose gas. We do not expect this to be a good approximation if the freeze-out point is near the critical point. The sigma field is the order parameter for the transition and near the critical point it therefore develops large critical long wavelength fluctuations. These fluctuations are responsible for singularities in thermodynamic quantities. We find that because of the $G\sigma\pi\pi$ coupling, the fluctuations of both the multiplicity and the mean transverse momentum of the charged pions do in fact diverge at the critical point.

We then estimate the size of the effects in a heavy ion collision. This requires first estimating the strength of the coupling constant G , and then taking into account the finite size of the system and the finite time during which the long wavelength fluctuations can develop. We find a large increase in the fluctuations of both the multiplicity and the mean transverse momentum of the pions. This increase would be divergent in the infinite volume limit precisely at the critical point. We apply finite size and finite time scaling to estimate how close the system created in a heavy ion collision can come to the critical singularity, and consequently how large an effect can be seen in the event-by-event fluctuations of the pions. We conclude that the nonmonotonic changes in the variance of the event-by-event fluctuation of the pion multiplicity and momenta which are induced by the universal physics characterizing the critical point can easily be between one and two orders of magnitude greater than the statistical errors in the present data.

The value of the coupling G in vacuum can be estimated either from the relationship between the sigma and pion masses and f_π or from the width of the sigma. Both yield an estimate $G \sim 1900$ MeV, where we have used $m_\sigma = 600$ MeV. The width of the sigma is so large that this “particle” is only seen as a broad bump in the s -wave $\pi - \pi$ scattering cross-section. The vacuum $\sigma\pi\pi$ coupling must be at least as large as $G \sim 1900$ MeV, since the sigma would otherwise be too narrow.

The vacuum value of G would not change much if one were to take the chiral limit $m \rightarrow 0$. The situation is different at the critical point. Taking the quark mass to zero while following the critical endpoint leads one to the tricritical point P in the phase diagram for QCD with two massless quarks. At

this point, G vanishes as we discuss below. This suggests that at E, the coupling G is less than in vacuum. In Ref. ¹, we use what we know about physics near the tricritical point P to make an estimate of how much the coupling G is reduced at the critical endpoint E (with the quark mass m having its physical value), relative to the vacuum value $G \sim 1900$ MeV estimated above.

We begin by recalling some known results. (For details, see Refs. ^{14,15,3}.) In QCD with two massless quarks, a spontaneously broken chiral symmetry is restored at finite temperature. This transition is likely second order and belongs in the universality class of $O(4)$ magnets in three dimensions. At zero T , various models suggest that the chiral symmetry restoration transition at finite μ is first order. Assuming that this is the case, one can easily argue that there must be a tricritical point P in the $T\mu$ phase diagram, where the transition changes from first order (at higher μ than P) to second order (at lower μ), and such a tricritical point has been found in a variety of models.^{14,15,20} The nature of this point can be understood by considering the Landau-Ginzburg effective potential for ϕ_α , order parameter of chiral symmetry breaking:

$$\Omega(\phi_\alpha) = \frac{a}{2}\phi_\alpha\phi_\alpha + \frac{b}{4}(\phi_\alpha\phi_\alpha)^2 + \frac{c}{6}(\phi_\alpha\phi_\alpha)^3. \quad (5)$$

The coefficients a , b and $c > 0$ are functions of μ and T . The second order phase transition line described by $a = 0$ at $b > 0$ becomes first order when b changes sign, and the tricritical point P is therefore the point at which $a = b = 0$. The critical properties of this point can be inferred from universality^{14,15}, and the exponents are as in the mean field theory (5). We will use this below. Most important in the present context is the fact that because $\langle\phi\rangle = 0$ at P, there is no $\sigma\pi\pi$ coupling, and $G = 0$ there.

In real QCD with nonzero quark masses, the second order phase transition becomes a smooth crossover and the tricritical point P becomes E, the second order critical endpoint of a first order phase transition line. Whereas at P there are four massless scalar fields undergoing critical long wavelength fluctuations, the σ is the only field which becomes massless at the point E, and the point E is therefore in the Ising universality class^{14,15}. The pions remain massive at E because of the explicit chiral symmetry breaking introduced by the quark mass m . Thus, when we discuss physics near E as a function of μ and T , but at fixed m , we will use universal scaling relations with exponents from the three dimensional Ising model. Our present purpose, however, is to imagine varying m while changing T and μ in such a way as to stay at the critical point E, and ask how large G (and m_π) become once m is increased from zero (the tricritical point P at which $G = m_\pi = 0$) to its physical value. For this task, we use exponents describing universal physics near P. Applying tricritical scaling

relations all the way up to a quark mass which is large enough that m_π is not small compared to T_c may introduce some uncertainty into our estimate.

We first determine the trajectory of the critical line of Ising critical points E as a function of quark mass m ,^e and then find that $G \sim m^{3/5}$ along this line, where m is the light quark mass. Thus the coupling G is suppressed compared to its “natural” vacuum value G_{vac} by a factor of order $(m/\Lambda_{\text{QCD}})^{3/5}$. Taking $\Lambda_{\text{QCD}} \sim 200$ MeV, $m \sim 10$ MeV we obtain our estimate

$$G_E \sim \frac{G_{\text{vac}}}{6} \sim 300 \text{ MeV} . \quad (6)$$

The main source of uncertainty in this estimate is our inability to compute the various nonuniversal masses which enter the estimate as prefactors in front of the m dependence which we have followed. In other words, we do not know the correct value to use for Λ_{QCD} in the suppression factor which we write as $(m/\Lambda_{\text{QCD}})^{3/5}$.

The final ingredient we need is an estimate of the correlation length ξ of the sigma field, which is infinite at the critical point. In practice, there are important restrictions on how large ξ can become. Two particle interferometry¹⁸ suggests that the size of regions over which freeze-out is homogeneous is roughly 12 fm in both the longitudinal and transverse directions. This means that the finite size of the system limits ξ to be less than about this value. The finite time restriction is stricter, but harder to estimate.²² Although the size of the system allows the correlation length to grow to 12 fm, there may not be enough time for such long correlations to grow. We use $\xi_{\text{max}} \sim 6$ fm as a rough estimate of the largest correlation length possible if control parameters are chosen in such a way that the system freezes out close to the critical point.

We now return to our discussion of the effects of the long wavelength sigma fluctuations on the fluctuations of the pions. We use mean field theory throughout Ref. ¹. The fluctuations of the sigma field around the minimum of $\Omega(\sigma)$ are not small; however, this does not make much difference to the quantities of interest, all of which diverge like $m_\sigma^{-2} \sim \xi^2$ at the critical point. The divergence is that of the sigma field susceptibility, and for the 3d-Ising universality class we know the corresponding exponent to be $\gamma/\nu = 2 - \eta$ which is ≈ 2 to within a few percent because η is small. We can therefore safely use mean-field mean field results with their m_σ^{-2} divergence, and will take $m_\sigma \sim 1/\xi_{\text{max}} \sim 1/(6 \text{ fm})$ in our estimates.

We now have all the ingredients in place to present our estimate of the size of the effect of the critical fluctuations of the sigma field on the fluctuations

^eSee Ref. ²¹ for a derivation of the analogous line of Ising points emerging from the tricritical point in the QCD phase diagram at zero μ as a function of m and the strange quark mass m_s . This tricritical point can be related to the one we are discussing by varying m_s .³

of the direct pions, via the coupling G . We express the size of the effect of interest by rewriting the ratio \sqrt{F} of (2) and (3) as

$$\sqrt{F} = \sqrt{F_B F_{\text{res}} F_{\text{EC}} F_{\sigma}}$$

and presenting F_{σ} . We find:

$$F_{\sigma} = 1 + 0.35 \left(\frac{G_{\text{freeze-out}}}{300 \text{ MeV}} \right)^2 \left(\frac{\xi_{\text{freeze-out}}}{6 \text{ fm}} \right)^2. \quad (7)$$

where we have taken $T = 120 \text{ MeV}$ and $\mu_{\pi} = 60 \text{ MeV}$. F_{σ} will be reduced by about a factor of two, because not all of the pions which are observed are direct. The coupling G transmits the effects of the critical σ fluctuations to the pions at freezeout, not to the (heavier) resonances. The size of the effect depends quadratically on the coupling G . We argued above that G is reduced to $G_E \sim 300 \text{ MeV}$ at the critical point. However, freeze-out may occur away from the critical point, in which case G would be larger, although still much smaller than its vacuum value. The size of the effect also depends quadratically on the sigma correlation length at freeze-out, and we have seen that there are many caveats in an estimate like $\xi_{\text{freeze-out}} \sim \xi_{\text{max}} \sim 6 \text{ fm}$.

We have studied two different effects of the critical fluctuations on \sqrt{F} . First, $F_{\text{EC}} \rightarrow 1$, leading to about a 1% increase in \sqrt{F} . The direct effect of the critical fluctuations is a much larger increase in \sqrt{F} by a factor of $\sqrt{F_{\sigma}}$. We have displayed the various uncertainties in the factors contributing to our estimate (7) so that when an experimental detection of an increase and then subsequent decrease in \sqrt{F} occurs, as control parameters are varied and the critical point is approached and then passed, we will be able to use the measured magnitude of this nonmonotonic effect to constrain these uncertainties. It should already be clear that an effect as large as 10% in $\sqrt{F_{\sigma}}$ is easily possible; this would be 50 times larger than the statistical error in the present data.

We now give a brief account of the effect of critical fluctuations on $\langle(\Delta N)^2\rangle$ and $\langle\Delta N \Delta p_T\rangle$. The contribution of the direct pions to $\langle(\Delta N)^2\rangle$ can easily double, but the multiplicity fluctuations are dominated by the pions from resonance decay, so we estimate that the critical multiplicity fluctuations lead to about a 10-20% increase in $\langle(\Delta N)^2\rangle$. (This neglects the pions from sigma decay. See below.) The cross-correlation $\langle\Delta N \Delta p_T\rangle$ only receives contributions from nontrivial effects, and we find that near the critical point, the contribution from the interaction with the sigma field is dominant. We estimate that (for $G_{\text{freeze-out}} \sim 300 \text{ MeV}$ and $\xi_{\text{freeze-out}} \sim 6 \text{ fm}$) the cross-correlation will be a factor of 10-15 times larger than in the absence of critical fluctuations! The

lesson is clear: although this correlation is small, it may increase in magnitude by a very large factor near the critical point.

The effects of the critical fluctuations can be detected in a number of ways. First, one can find a nonmonotonic increase in F_σ , the suitably normalized increase in the variance of event-by-event fluctuations of the mean transverse momentum. Second, one can find a nonmonotonic increase in $\langle(\Delta N)^2\rangle$. Both these effects can easily be between one and two orders of magnitude greater than the statistical errors in present data. Third, one can find a nonmonotonic increase in the magnitude of $\langle\Delta p_T \Delta N\rangle$. This quantity is small, and it has not yet been demonstrated that it can be measured. However, it may change at the critical point by a large factor, and is therefore worth measuring. In addition to effects on these and many other observables, it is perhaps most distinctive to measure the microscopic correlator $\langle\Delta n_p \Delta n_k\rangle$ itself. The effects proportional to $1/m_\sigma^2$ in has a specific dependence on p and k . It introduces off-diagonal correlations in pk space. Like the off-diagonal anti-correlation introduced by energy conservation, this makes it easy to distinguish from the Bose enhancement effect, which is diagonal in pk . Near the critical point, the off-diagonal anti-correlation vanishes and the off-diagonal correlation due to sigma exchange grows. Furthermore, the effect of σ exchange is not restricted to identical pions, and should be visible as correlations between the fluctuations of π^+ and π^- . The dominant diagonal term proportional to δ_{pk} will be absent in the correlator $\langle\Delta n_p^+ \Delta n_k^-\rangle$, and the effects of σ exchange will be the dominant contribution to this quantity near the critical point.

4 Pions From Sigma Decay

Having analyzed the effects of the sigma field on the fluctuations of the direct pions, we next ask what becomes of the sigmas themselves. For choices of control parameters such that freeze-out occurs at or near the critical endpoint, the excitations of the sigma field, sigma (quasi)particles, are nearly massless at freeze-out and are therefore numerous. Because the pions are massive at the critical point, these σ 's cannot immediately decay into two pions. Instead, they persist as the temperature and density of the system further decrease. During the expansion, the in-medium σ mass rises towards its vacuum value and eventually exceeds the two pion threshold. Thereafter, the σ 's decay, yielding a population of pions which do not get a chance to thermalize because they are produced after freeze-out. We estimate the momentum spectrum of these pions produced by delayed σ decay. An event-by-event analysis is not required in order to see these pions. The excess multiplicity at low p_T will appear and then disappear in the single particle inclusive distribution as

control parameters are varied such that the critical point is approached and then passed.

In calculating the inclusive single-particle p_T -spectrum of the pions from sigma decay, we must treat m_σ as time-dependent, and should also take G to evolve with time. However, the dominant time-dependent effect is the opening up of the phase space for the decay as m_σ increases with time and crosses the two-pion threshold. We therefore treat G as a constant. We have estimated that in vacuum with $m_\sigma = 600$ MeV, the coupling is $G \sim 1900$ MeV, whereas at the critical end point with $m_\sigma = 0$, the coupling is reduced, perhaps by as much as a factor of six or so. In this section, we need to estimate G at the time when m_σ is at or just above twice the pion mass. We will use $G \sim 1000$ MeV, recognizing that we may be off by as much as a factor of two.

We parametrize the time dependence of the sigma mass by $m_\sigma(t) = 2m_\pi(1 + t/\tau)$, where we have defined $t = 0$ to be the time at which m_σ has risen to $2m_\pi$ and have introduced the timescale τ over which m_σ increases from $2m_\pi$ to $4m_\pi$. It seems likely that $5 \text{ fm} < \tau < 20 \text{ fm}$. We find that the mean transverse momentum of the pions produced by sigma decay is

$$\langle p_T \rangle \sim 0.58 m_\pi \left(\frac{1000 \text{ MeV}}{G} \right)^{2/3} \left(\frac{10 \text{ fm}}{\tau} \right)^{1/3}. \quad (8)$$

We therefore estimate that if freeze-out occurs near the critical point, there will be a nonthermal population of pions with transverse momenta of order half the pion mass with a momentum distribution given in Ref. ¹.

How many such pions can we expect? This is determined by the sigma mass at freeze-out. If m_σ is comparable to m_π at freeze-out, then there are half as many σ 's at freeze-out as there are charged pions. Since each sigma decays into two pions, and two thirds of those pions are charged, the result is that the number of charged pions produced by sigma decays after freeze-out is 2/3 of the number of charged pions produced directly by the freeze-out of the thermal pion gas. Of course, if freeze-out occurs closer to the critical point at which m_σ can be as small as $(6 \text{ fm})^{-1}$, there would be even more sigmas. We therefore suggest that as experimenters vary the collision energy, one way they can discover the critical point is to see the appearance and then disappearance of a population of pions with $\langle p_T \rangle \sim m_\pi/2$ which are almost as numerous as the direct pions. Yet again, it is the nonmonotonicity of this signature as a function of control parameters which makes it distinctive.

The event-by-event fluctuations of the multiplicity of these pions reflect the fluctuations of the sigma field whence they came³. We estimate¹ that the event-by-event fluctuations of the multiplicity of the pions produced in sigma decay will be $\langle (\Delta N)^2 \rangle \approx 2.74 \langle N \rangle$. We have already seen in that the critical

fluctuations of the sigma field increase the fluctuations in the multiplicity of the direct pions sufficiently that the increase in the fluctuation of the multiplicity of all the pions will be increased by about 10 – 20%. We now see that in the vicinity of the critical point, there will be a further nonmonotonic rise in the fluctuations of the multiplicity of the population of pions with $\langle p_T \rangle \sim m_\pi/2$ which are produced in sigma decay.

5 Outlook

Our understanding of the thermodynamics of QCD will be greatly enhanced by the detailed study of event-by-event fluctuations in heavy ion collisions. We have estimated the influence of a number of different physical effects, some special to the vicinity of the critical point but many not. The predictions of a simple resonance gas model, which does not include critical fluctuations, are to this point in very good agreement with the data. More detailed study, for example with varying cuts in addition to new observables, will help to further constrain the nonthermodynamic fluctuations, which are clearly small, and better understand the different thermodynamic effects. The signatures we analyze allow experiments to map out distinctive features of the QCD phase diagram. The striking example which we have considered in detail is the effect of a second order critical end point. The nonmonotonic appearance and then disappearance of any one of the signatures of the critical fluctuations which we have described would be strong evidence for the critical point. Furthermore, if a nonmonotonic variation is seen in several of these observables, then the maxima in all the signatures must occur simultaneously, at the same value of the control parameters. Simultaneous detection of the effects of the critical fluctuations on different observables would turn strong evidence into an unambiguous discovery.

Acknowledgments

We are grateful to G. Roland for providing us with preliminary NA49 data. We acknowledge helpful conversations with M. Creutz, U. Heinz, M. Gaździcki, V. Koch, St. Mrówczyński, G. Roland and T. Trainor.

I thank the organizers of SEWM'98 for a conference which, by bringing together those studying QCD matter in extreme conditions and those studying electroweak matter in extreme conditions, was stimulating and enjoyable.

This work was supported in part by a DOE Outstanding Junior Investigator Award, by the A. P. Sloan Foundation, and by the DOE under cooperative research agreement DE-FC02-94ER40818.

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